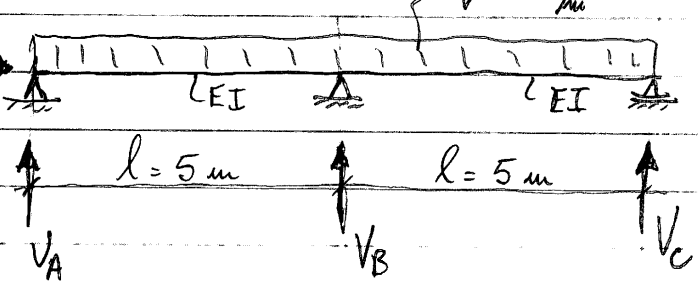
Voorbeeld Krachtenmethode

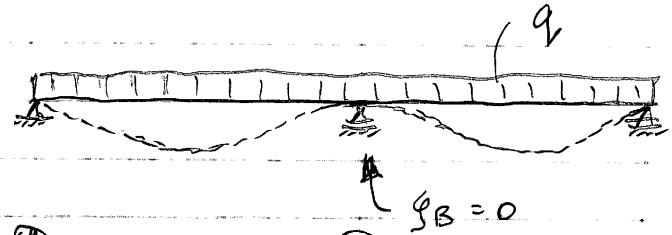
- symmetrische ligger (l = constant)
- belasting q gelijk (links en rechts)



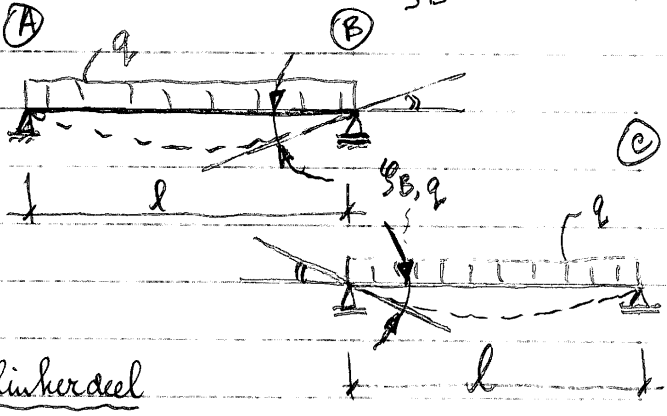
gevraagd
krachtsverdeling in ligger?

oplossing
$n = R - E = 4 - 3 = 1$ → ligger enkelvoudig statisch onbepaald.

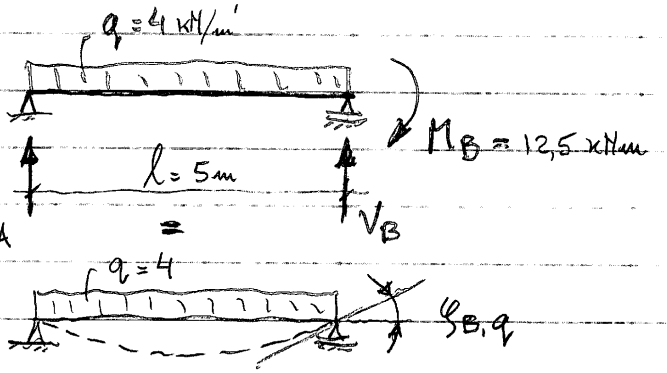
kies M_B als statisch onbepaalde
eis: $\varphi_B = 0$
(volgt uit symmetrie ligger en symmetrie belasting)



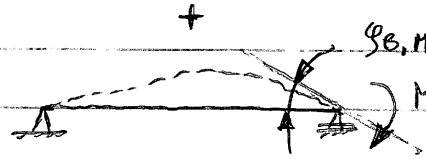
werkwijze
- ligger doorsnijden in B
- 2 statisch bepaalde liggers
- los op ligger A-B
- bepaal $\varphi_{B,q}$ tgv. q
- breng aan moment in B
- bepaal φ_{B,M_B} tgv. M_B
- stel op $\varphi_{B,q} + \varphi_{B,M_B} = 0$



linkerdeel



+



$$\varphi_{B,q} = \frac{4 \times 5^3}{24 \cdot EI} = \frac{500}{24\,EI} \ (\curvearrowleft)$$

$$\varphi_{B,M_B} = \frac{M_B \times 5}{3 \cdot EI} \ (\curvearrowright)$$

$$\boxed{M_B = 12{,}5 \text{ kN.m}}$$

ligger A-B

$\Sigma M_{tov.B} = 0$:
$V_A \times 5 - (4 \cdot 5) \times 2{,}5 + 12{,}5 = 0 \rightarrow \boxed{V_A = 7{,}5 \text{ kN } (\uparrow)}$

$\Sigma V = 0$
$7{,}5 - (4 \cdot 5) + V_B = 0 \rightarrow V_B = 12{,}5 \text{ kN } (\uparrow)$

ligger B-C

$\boxed{V_C = 7{,}5 \text{ kN } (\uparrow)}$

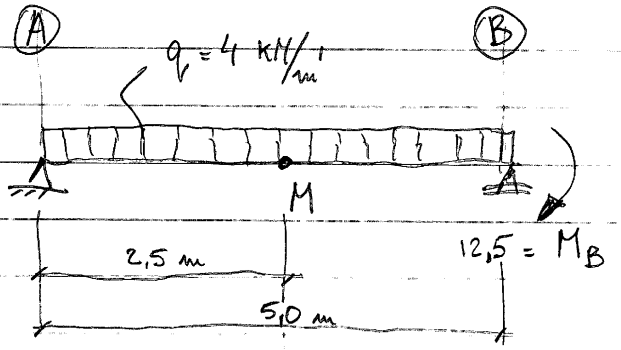
$V_B = 12{,}5 \text{ kN } (\uparrow)$

$\boxed{V_{B,TOTAAL} = 25 \text{ kN } (\uparrow)}$

D-lijn
M-lijn
δ-lijn

zie blad (6)

Doorbuiging m.b.v. V.G.M.M.'s



gevraagd

doorbuiging van punt M: δ_M ?

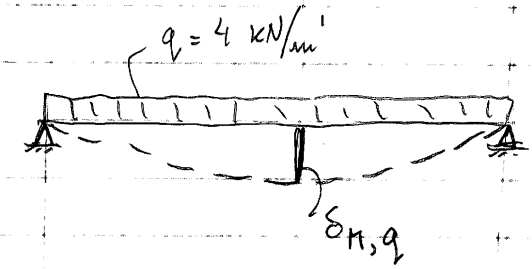
profiel HE 100 A

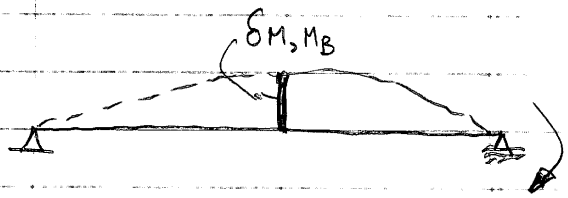
oplossing

δ_M uitrekenen voor 2 belastinggevallen

① tgv. belasting q (↓)

② tgv. belasting M_B (↑)





$M_B = 12{,}5$ kNm

$$① \quad \delta_{M,q} = \frac{5\,q\,l^4}{384\,E\,I}$$

$$② \quad \delta_{M,M_B} = \frac{M_B\,l^2}{16\,E\,I}$$

} in N en mm

$q = 4$ kN/m' $= 4$ N/mm'

$l = 5$ m $= 5000$ mm

$M_B = 12{,}5$ kNm $= 12{,}5 \times 10^6$ N·mm

$E = 210\,000$ N/mm^2

$I = 3490 \times 10^3$ mm^4

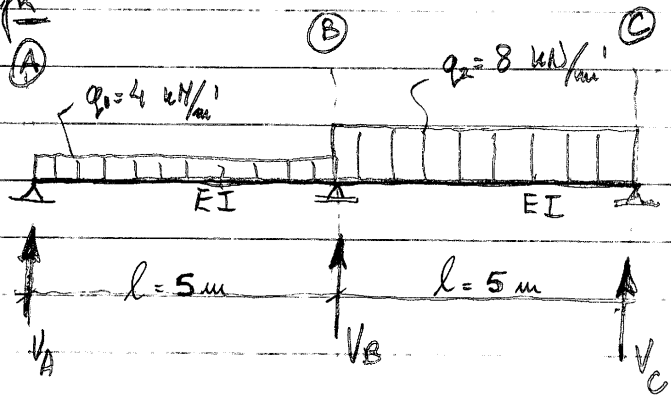
$$\delta_M = \delta_{M,q} + \delta_{M,M_B}$$

$$\delta_M = \frac{5 \times 4 \times 5000^4}{384 \times 210\,000 \times 3490 \times 10^3}\,(↓) - \frac{12{,}5 \times 10^6 \times 5000^2}{16 \times 210\,000 \times 3490 \times 10^3}\,(↑)\ \text{mm}$$

$$\delta_M = 44{,}4 - 26{,}6$$

$$\boxed{\delta_M = 17{,}8\ \text{mm}\ (↓)}$$ [controle MATRIX FRAME OK]

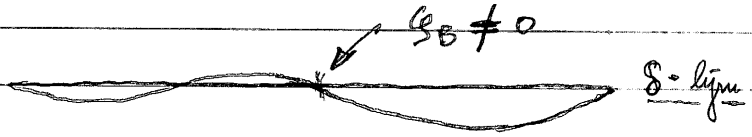
Symmetrische ligger, belasting ongelijk



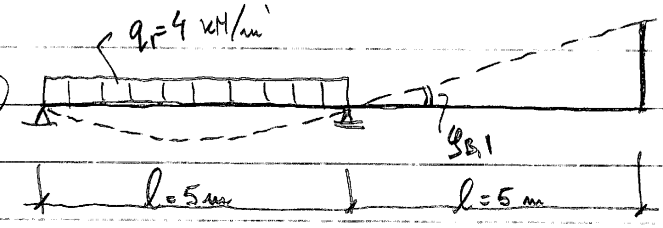
gevraagd
Krachtsverdeling in ligger ?

oplossingen
① vervormingsmethode
② krachtenmethode

① ligger is 1 - S.O.
kies V_C als statisch onbepaalde
dis: $\delta_C = 0$

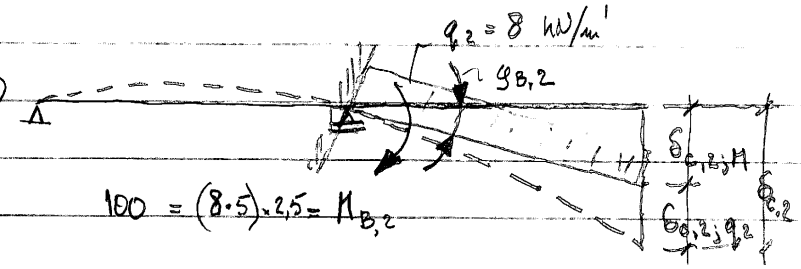


werkwijze
haal steunpunt C weg, bepaal doorbuiging in C, breng kracht V_C aan, bepaal doorbuiging in C.
bepaal verschillende belastinggevallen en
$\delta_{C,1} + \delta_{C,2} + \delta_{C,3} = 0$



① $\delta_{C,1} = \varphi_{B,1} \times 5$

$\delta_{C,1} = \frac{4 \times 5^3}{24\,EI} \times 5 \rightarrow \boxed{\delta_{C,1} = \frac{104,17}{EI}\ (\uparrow)}$

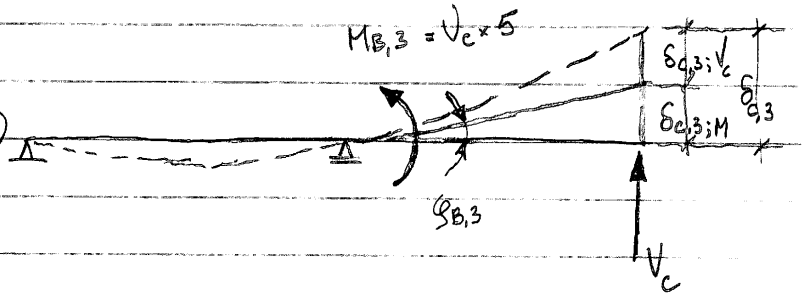


② $\delta_{C,2} = \delta_{C,2;M} + \delta_{C,2;q_2}$

$\delta_{C,2} = \varphi_{B,2} \times 5 + \delta_{C,2;q_2}$

$\delta_{C,2} = \frac{100 \times 5}{3\,EI} \times 5 + \frac{8 \times 5^4}{8\,EI}$

$\rightarrow \boxed{\delta_{C,2} = \frac{1458,33}{EI}\ (\downarrow)}$

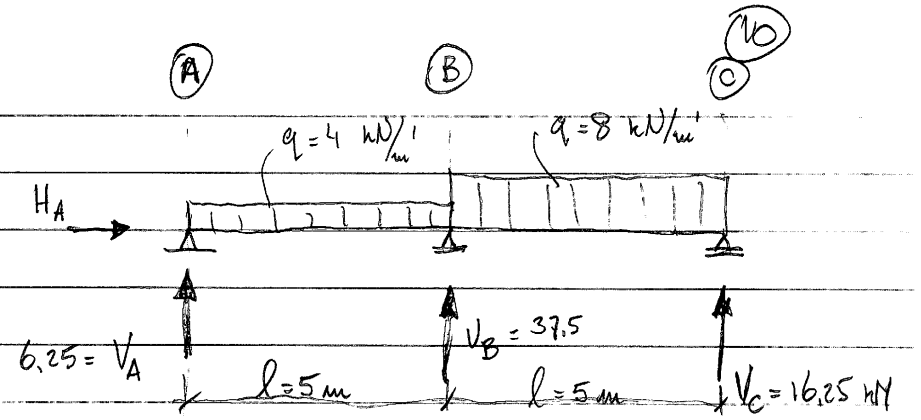


③ $\delta_{C,3} = \delta_{C,3;M} + \delta_{C,3;V_C} = \varphi_{B,3} \times 5 + \delta_{C,3;V_C} = \frac{(V_C \times 5) \times 5}{3EI} \times 5 + \frac{V_C \times 5^3}{3\,EI}$

$\rightarrow \boxed{\delta_{C,3} = \frac{V_C \times 83,33}{EI}\ (\uparrow)}$

$\frac{104,17}{EI}(\uparrow) - \frac{1458,33}{EI}(\downarrow) + \frac{V_C \times 83,33}{EI}(\uparrow) = 0$

$\rightarrow \boxed{V_C = 16,25\ \text{kN}\ (\uparrow)}$



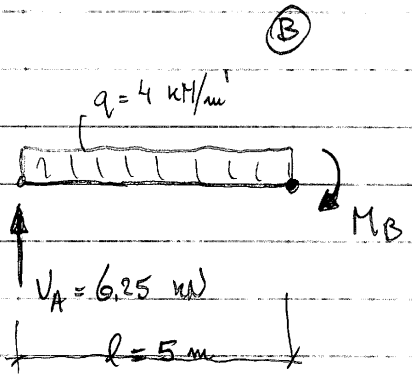
$\Sigma M_{tov.A} = 0$: $-V_B \times 5 - 16{,}25 \times 10 + (4 \times 5) \times 2{,}5 + (8 \times 5) \times 7{,}5 = 0$

$\boxed{V_B = 37{,}5 \text{ kN } (\uparrow)}$

$\Sigma V = 0$: $\boxed{V_A = 6{,}25 \text{ kN } (\uparrow)}$

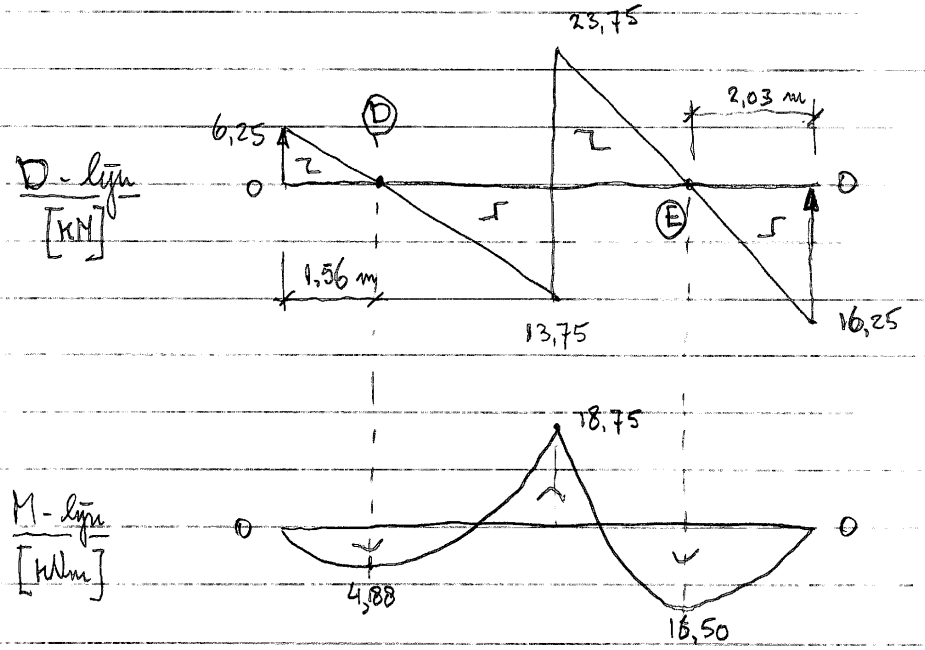
$\Sigma H = 0$: $\boxed{H_A = 0 \text{ kN}}$

Hoe groot is moment in punt B ?



$6{,}25 \times 5 - (4 \times 5) \times 2{,}5 + M_B = 0$

$\boxed{M_B = 18{,}75 \text{ kNm } (\curvearrowright)}$

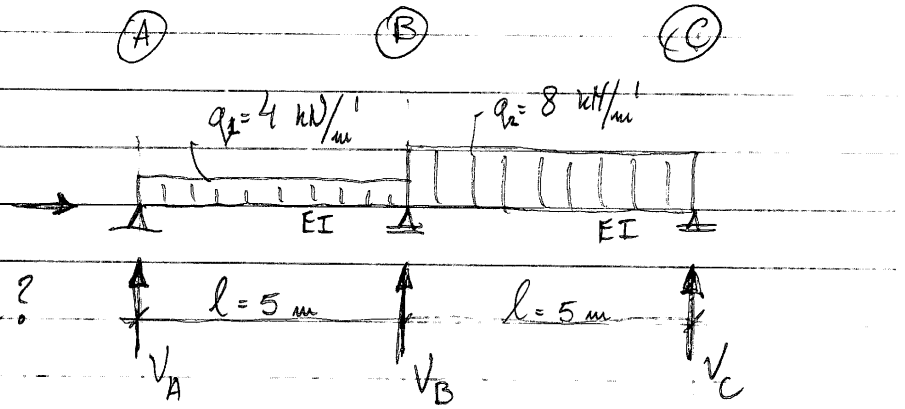


$M_D = (6{,}25 \times 1{,}56) \times \frac{1}{2} \rightarrow M_D = 4{,}88 \text{ kNm } (\cup)$

$M_E = (16{,}25 \times 2{,}03) \times \frac{1}{2} \rightarrow M_E = 16{,}50 \text{ kNm } (\cup)$

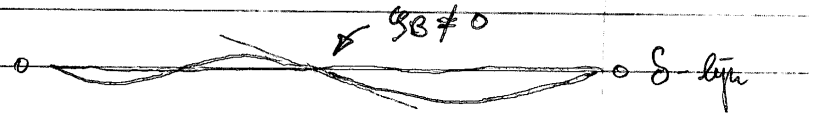
② Krachtenmethode

- ligger symmetrisch
- belasting ongelijk



gevraagd: krachtsverdeling in ligger?

kies φ_B als statisch onbepaalde

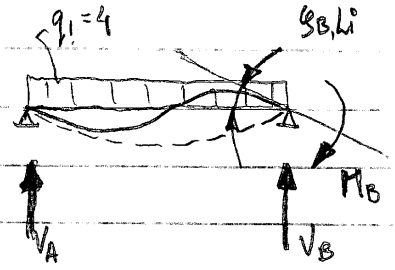


$\varphi_{B,Li} = -\varphi_{B,q_1}(\curvearrowright) + \varphi_{B,M_B}(\curvearrowright)$

$-\frac{q_1 \cdot l^3}{24EI} + \frac{M_B \cdot l}{3EI}$

$-\frac{4 \cdot 5^3}{24EI} + \frac{M_B \cdot 5}{3EI}$

$\varphi_{B,Li} = -\frac{500}{24EI} + \frac{M_B \cdot 5}{3EI}$

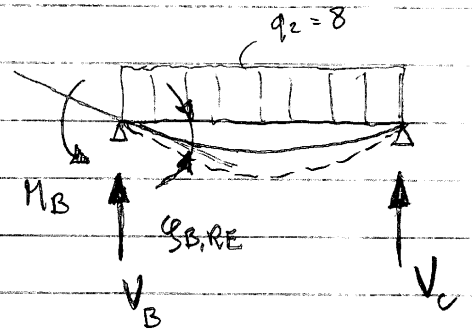


$\varphi_{B,RE} = \varphi_{B,q_2}(\curvearrowright) - \varphi_{B,M_B}(\curvearrowleft)$

$\frac{q_2 \times l^3}{24EI} - \frac{M_B \times 5}{3EI}$

$\frac{8 \times 5^3}{24EI} - \frac{M_B \cdot 5}{3EI}$

$\varphi_{B,RE} = \frac{1000}{24EI} - \frac{M_B \times 5}{3EI}$



$\varphi_{B,Li} = \varphi_{B,RE}$

$-\frac{500}{24EI} + \frac{5 \cdot M_B}{3EI} = \frac{1000}{24EI} - \frac{5M_B}{3EI}$

$\frac{10}{3} M_B = \frac{1500}{24}$

$\boxed{M_B = 18{,}75 \text{ kNm}}$

D- en M-lijnen tekenen

deel A-B

$\Sigma M_B = 0$: $V_A \times 5 - (4 \cdot 5) \times 2{,}5 + 18{,}75 = 0$

$\boxed{V_A = 6{,}25 \text{ kN } (\uparrow)}$

$\Sigma V = 0$: $V_B = 13{,}75 \text{ kN } (\uparrow)$

deel B-C

$\Sigma M_B = 0$: $-V_C \times 5 + (8 \cdot 5) \times 2{,}5 - 18{,}75 = 0$

$\boxed{V_C = 16{,}25 \text{ kN } (\uparrow)}$

$\Sigma V = 0$: $V_B = 23{,}75 \text{ kN } (\uparrow)$

$\boxed{V_{B,Tot} = 37{,}50 \text{ kN } (\uparrow)}$