



Week 09 - ribWBKII

Theorie: Integraalrekening

Onderwerp: Kromming, zakking en hoekverandering (vervolg)

Opgave#1

Gegeven

Ligger op 2 steunpunten met puntlast

Lengte = 6 meter

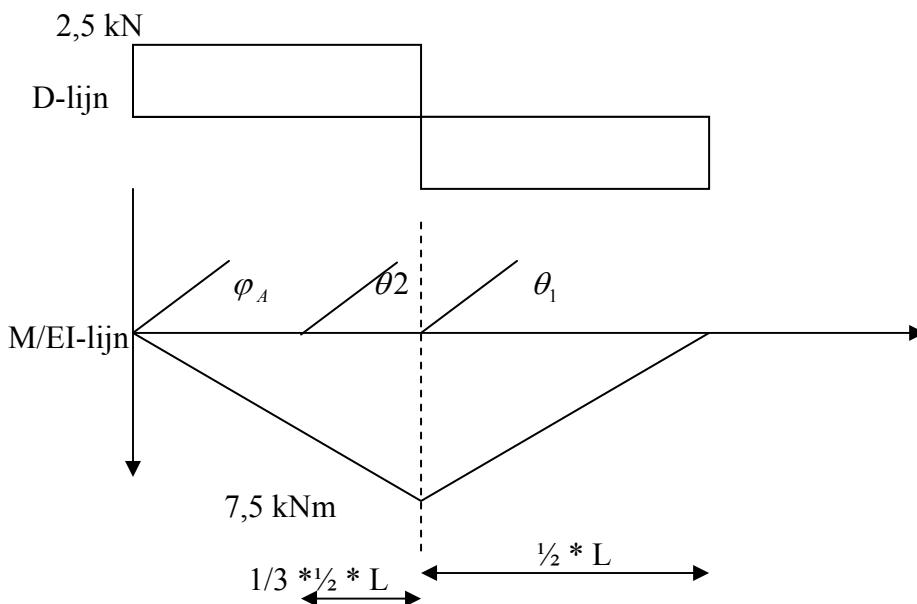
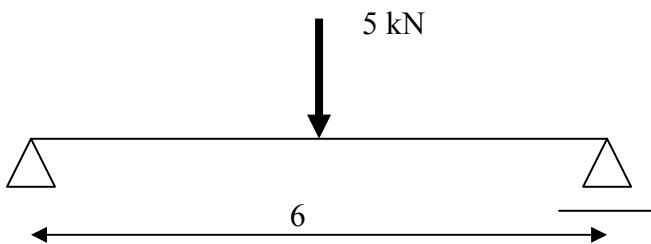
Kracht = 5 kN

$E = 210.000 \text{ N/mm}^2$

$I_y = 934 * 10^4 \text{ mm}^4$

Gevraagd

01. Dwarskrachtenlijn
02. Momentenlijn
03. Hoekverandering
04. Zakking
05. Zakkingslijn



Randvoorwaarden: zakking in A en B is nul

Knikje positief dus zakking is negatief

$$\text{oppervlakte} = \frac{1}{2} * \frac{1}{4} Fl * l = \frac{Fl^2}{8}$$

$$\theta_1 = \frac{\frac{1}{2} * \frac{1}{4} Fl * \frac{1}{2} l}{EI} = \frac{Fl^2}{16EI} = \frac{5 * 36}{16EI} = \frac{11,25}{EI} = \frac{11,25}{1961,4} = 0.0057$$

Bepalen van de hoekverdraaiing (knikje positief zakking negatief)

$$\omega_B = 0$$

2^e Stelling

$$\omega_B = -\varphi_A * l - \theta_1 * \frac{1}{2} l \Rightarrow \varphi_A = -\frac{\theta_1 * \frac{1}{2} l}{l} = -\frac{Fl^2}{16EI} = -\frac{5 * 36}{16EI} = -\frac{11,25}{EI} = -\frac{11,25}{1961,4} = -0.057$$

$$\varphi_B = -\varphi_A = 0.057$$

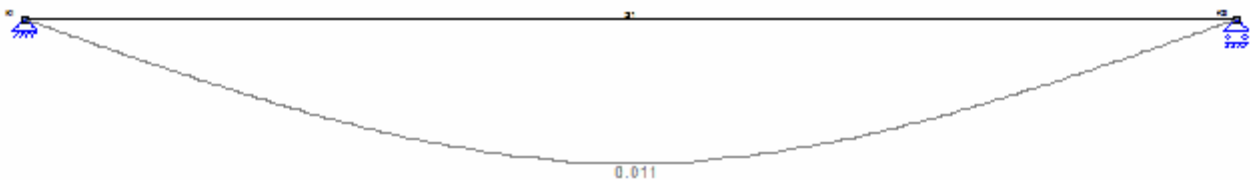
Zakking in het midden

Hiervoor moet het statisch moment van het gereduceerd oppervlak tussen A en het midden worden bepaald.

$$\theta_2 = \frac{Fl^2}{8EI} = \frac{5 * 36}{8EI} = \frac{22,5}{EI} = \frac{22,5}{1961,4} = 0.0115$$

$$\omega_{midden} = -\varphi_A * \frac{1}{2} l - \theta_2 * \frac{1}{6} l = -\frac{Fl^2 * \frac{1}{2} l}{16EI} - \frac{Fl^2 * \frac{1}{6} l}{16EI} = \frac{Fl^3}{48EI} = \frac{5 * 6^3}{48EI} = \frac{22,5}{EI}$$

$$\omega_{midden} = \frac{22,5}{1961,4} = 0,0115m = 11,5mm$$



Opgave#2

Gegeven

Ligger op twee steunpunten met puntlast

Lengte = 6 meter

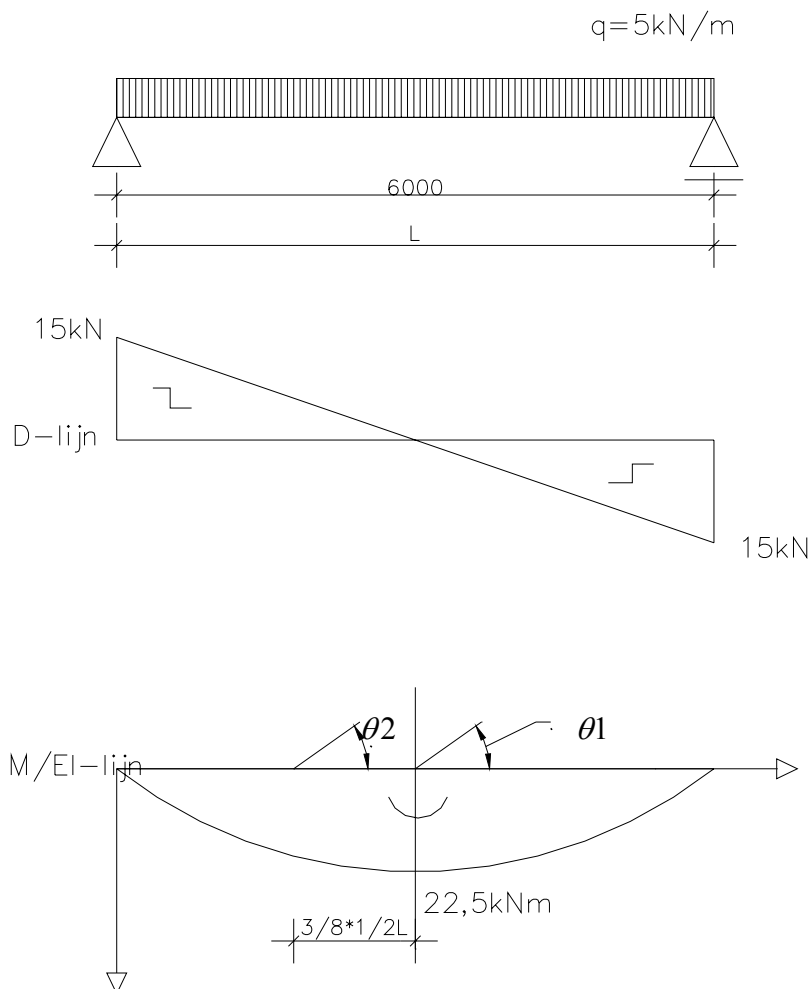
Kracht = 5 kN

$E = 210.000 \text{ N/mm}^2$

$I_y = 934 * 10^4 \text{ mm}^4$

Gevraagd

01. Dwarskrachtenlijn
02. Momentenlijn
03. Hoekverandering
04. Zakking
05. Zakkingslijn





$$V(x) = \int -q dx = -5x + C$$

$$M(x) = \int (-5x + c) dx = -2\frac{1}{2}x^2 + Cx + D$$

$$M(0) \rightarrow D=0$$

$$M(l) = 0 \Rightarrow 0 = -2\frac{1}{2}l^2 + Cl \Rightarrow C = 2\frac{1}{2}l$$

$$V(x) = -5x + 2\frac{1}{2}l$$

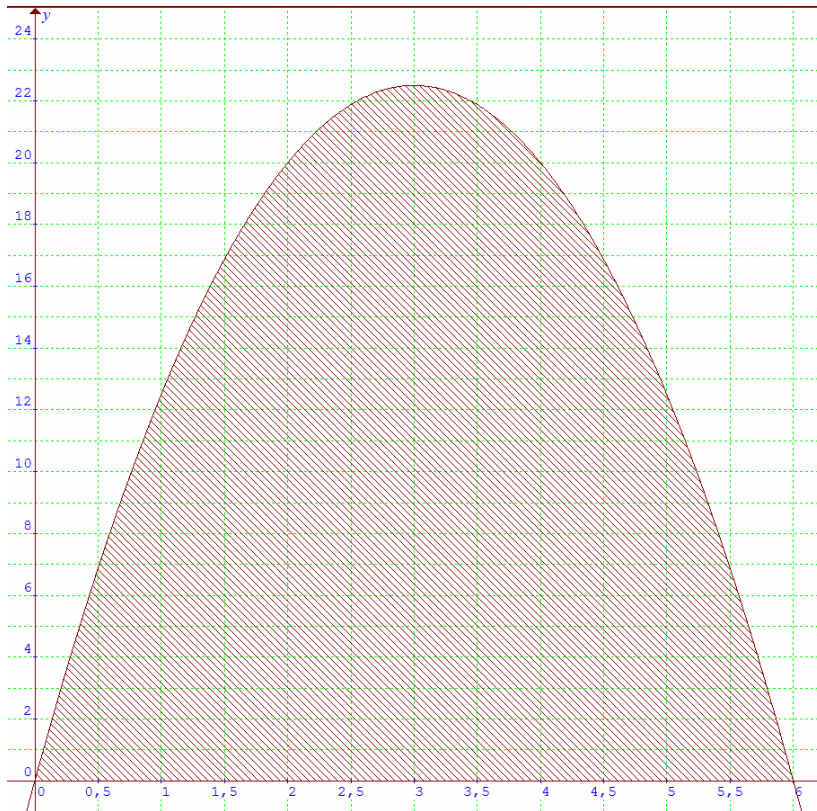
$$\text{Voor } x=0 \rightarrow V(0) = 15 \text{ kN}$$

$$\text{Voor } x=6 \rightarrow V(6) = -15 \text{ kN}$$

$$M(x) = -2\frac{1}{2}x^2 + \frac{1}{2}lx$$

$$\text{Voor } x=3 \rightarrow M(3) = 22,5 \text{ kNm}$$

Oppervlakte grafiek



$$f(x) = -2.5(x-3)^2 + 22.5 = -2.5x^2 + 15x$$



$$\text{Oppervlakte} = \int_0^6 (-2.5x^2 + 15x) dx = \left[-\frac{1}{1.2}x^3 + 7.5x^2 \right]_0^6 = -180 + 270 = 90$$

Of via de methode van het gereduceerde momentenvlak:

$$\text{Oppervlakte} = \frac{2}{3} * \frac{1}{8} ql^2 * l = \frac{1}{12} ql^3 = \frac{1}{12} * 5 * 6^3 = 90$$

$$\theta_1 = \frac{ql^3}{12EI}$$

Zakking in A en B is nul

Knikje naar boven dus zakking negatief.

$$\omega_B = -\varphi_A * l - \theta_1 * \frac{1}{2}l \Rightarrow \varphi_A = -\frac{ql^3}{24EI}$$

$$-\varphi_A = \varphi_B = \frac{ql^3}{24EI}$$

Zakking in het midden

Bepaling afstand (a) vanaf het zwaartepunt van het linkerdeel van de momentoppervlakte t.o.v. het midden van de totale momentoppervlakte.

$$O_{0,3} = \left[-\frac{1}{1.2}x^3 + 7.5x^2 \right]_0^3 = 45$$

$$S_y = \int_0^3 (-2.5x^2 + 15x) x dx = \int_0^3 (-2.5x^3 + 15x^2) dx = \left[-\frac{1}{1.6}x^4 + 5x^3 \right]_0^3 = 84.375$$

$$x = \frac{S_y}{A} = \frac{84.375}{45} = 1.875 \Rightarrow a = 3 - 1.875 = 1.125$$

$$\text{a is ook gelijk aan: } \frac{3}{8} * \frac{1}{2}l = \frac{3}{16}l = \frac{3}{16} * 6 = 1.125$$

$$\theta_2 = \frac{\frac{1}{8}ql^2 * \frac{1}{2}l * \frac{2}{3}}{EI} = \frac{2}{48} * ql^3 = \frac{ql^3}{24EI}$$

Knikje naar boven dus zakking negatief.

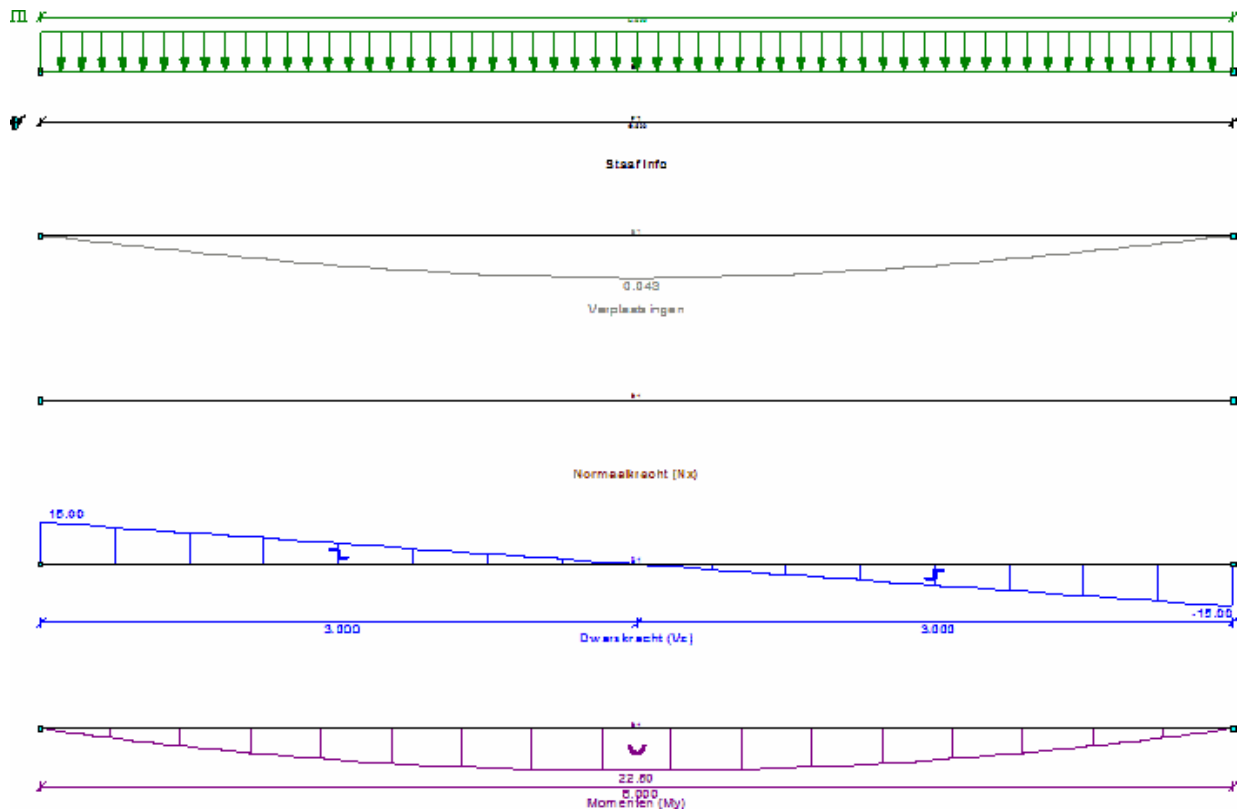
$$\omega_{midden} = -\varphi_A * \frac{1}{2}l - \theta_2 * \frac{3}{8} * \frac{1}{2}l$$

$$\omega_{midden} = \frac{ql^3}{24EI} * \frac{1}{2}l - \frac{ql^3}{24EI} * \frac{3}{16}l$$

$$\omega_{midden} = \frac{ql^4}{48EI} - \frac{3ql^4}{384EI} = \frac{5ql^4}{384EI}$$

$$\omega_{midden} = \frac{5 * 5 * 6^4}{384 * 1961,4} = 0.043m = 43mm$$

In Matrix-frame





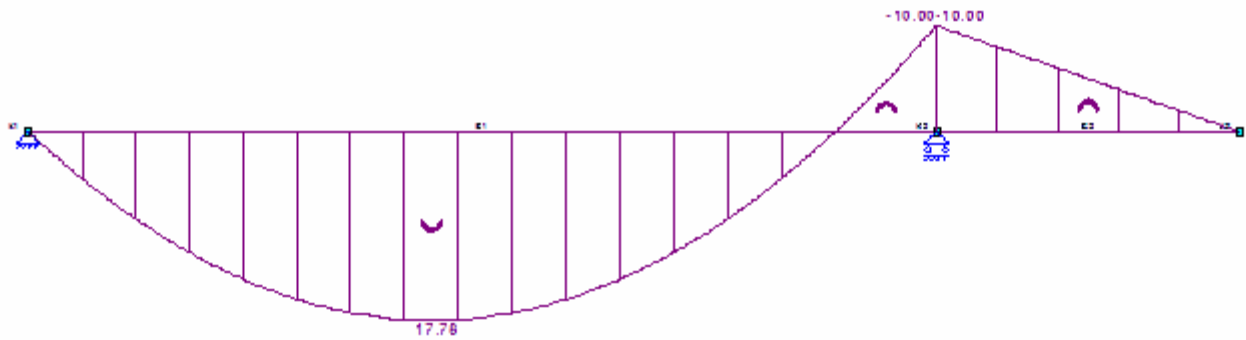
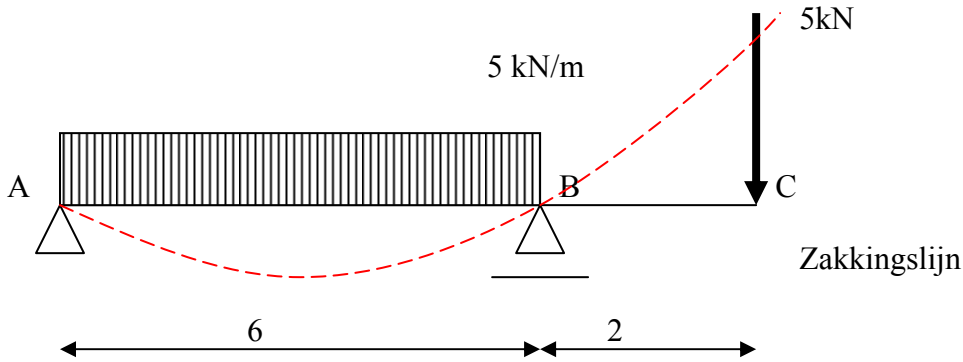
In Excel



Opgave#3

$E=210000 \text{ N/mm}^2$

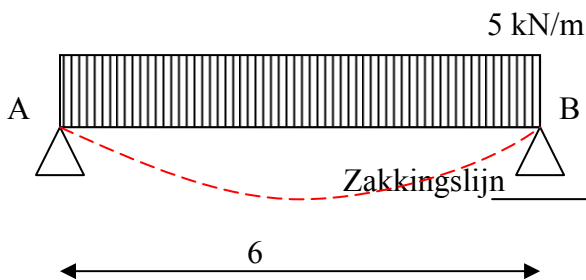
$I=934 * 10^4 \text{ mm}^4$



Oplossing

Toepassen van de "Vergeet-Mij-Nietjes". (zie bijlage)

Deel A-B





Hoekverandering in B

$$\varphi_B = \frac{ql^3}{24EI} = \frac{5 * 6^3}{24 * 1961,4} = 0,023rad$$

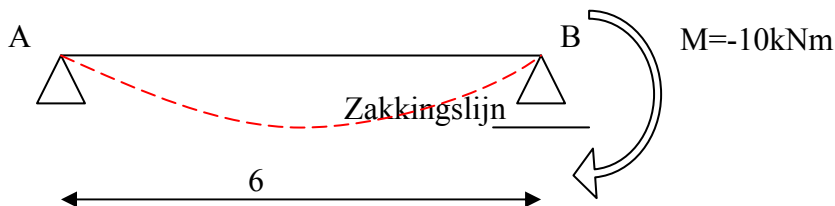
Zakking in het midden

$$\omega_{midden} = -\frac{5ql^4}{384EI} = \frac{5 * 5 * 6^4}{384 * 1961,4} = 0,043m = -43mm$$

Moment op B door kracht 5 kN

$$-5kN * 2m + M = 0 \Rightarrow M = 10kNm$$

Deel A-B



Hoekverandering in B

$$\varphi_B = \frac{Ml}{3EI} = \frac{-10 * 6}{3 * 1961,4} = -0,01rad$$

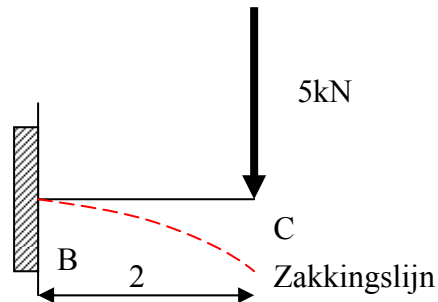
Zakking in midden door Moment

$$\omega_{midden} = \frac{Ml^2}{16EI} = -\frac{10 * 36}{16 * 1961,4} = 0,011m = 11mm$$

Totale zakking in het midden door gelijkmatige belasting en het Moment t.b.v. puntlast

$$\omega_{midden.totaal} = -43 + 11 = -32mm$$

Deel B-C



Totale hoekverdraaiing in B

$$\varphi_B = 0,023 - 0,01 = 0,013 \text{ rad}$$

Zakking in C

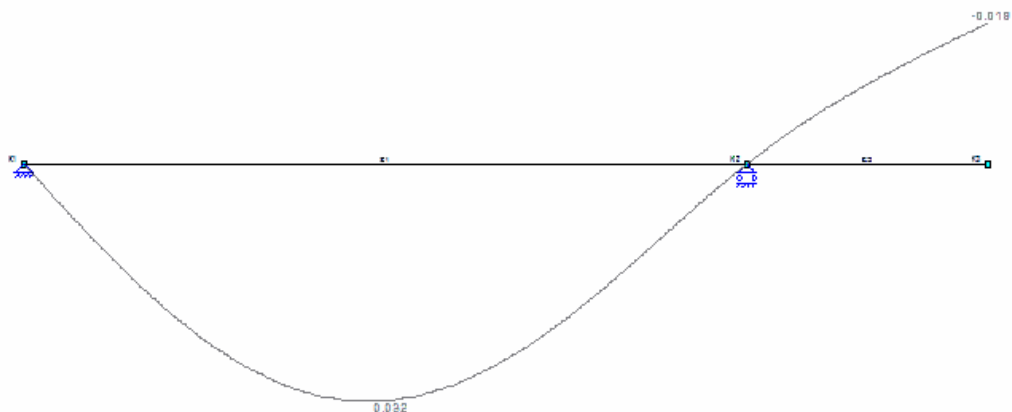
$$\omega_C = \frac{Fl^3}{3EI} = \frac{5 * 2^3}{3 * 1961,4} = 0,007 \text{ m} = -7 \text{ mm}$$

Statisch moment t.o.v. C

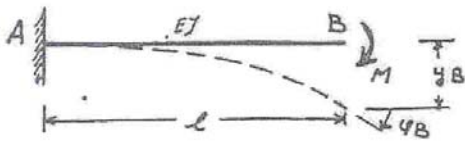
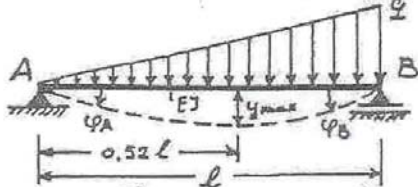
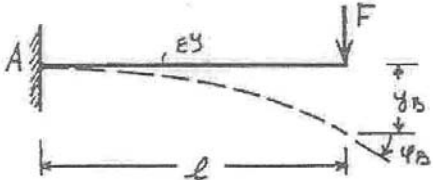
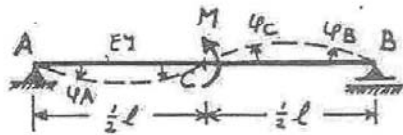
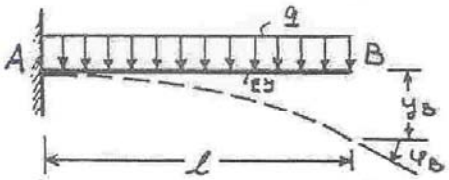
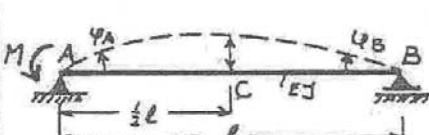
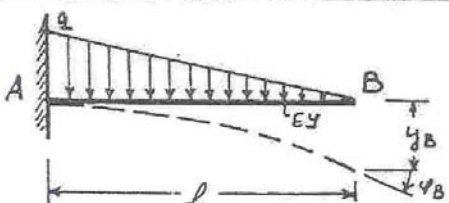
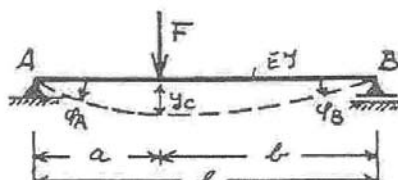
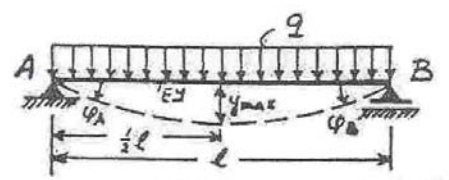
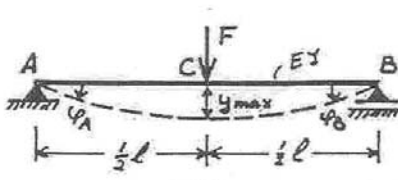
$$\varphi_B * l = 0,013 * 2 = 0,026 \text{ m} = 26 \text{ mm}$$

Zakking in C

$$= 26 - 7 = 19 \text{ mm}$$



Bijlage

<p>①</p>  <p>$\varphi_B = \frac{M \cdot l}{EJ}$; $y_B = \frac{M \cdot l^2}{2EJ}$</p>	<p>⑥</p>  <p>$\varphi_A = \frac{7ql^3}{360EJ}$; $\varphi_B = -\frac{8ql^3}{360EJ}$ $y_{max} = \frac{0,00659ql^4}{EJ}$</p>
<p>②</p>  <p>$\varphi_B = \frac{F \cdot l^2}{2EJ}$; $y_B = \frac{F \cdot l^3}{3EJ}$</p>	<p>⑦</p>  <p>$\varphi_A = \varphi_B = \frac{M \cdot l}{24EJ}$; $\varphi_C = -\frac{M \cdot l}{12EJ}$</p>
<p>③</p>  <p>$\varphi_B = \frac{ql^3}{6EJ}$; $y_B = \frac{ql^4}{8EJ}$</p>	<p>⑧</p>  <p>$\varphi_A = -\frac{M \cdot l}{3EJ}$; $\varphi_B = \frac{M \cdot l}{6EJ}$ $\varphi_C = \frac{M \cdot l}{24EJ}$; $y_C = \frac{M \cdot l^2}{16EJ}$</p>
<p>④</p>  <p>$\varphi_B = \frac{ql^3}{24EJ}$; $y_B = \frac{ql^4}{30EJ}$</p>	<p>⑨</p>  <p>$\varphi_A = \frac{F \cdot a \cdot b}{6lEJ} (l+b)$; $\varphi_B = -\frac{F \cdot a \cdot b}{6lEJ} (l+a)$ $y_C = \frac{F \cdot a^2 \cdot b^2}{32EJ}$</p>
<p>⑤</p>  <p>$\varphi_A = -\varphi_B = \frac{ql^3}{24EJ}$; $y_{max} = \frac{5ql^4}{384EJ}$</p>	<p>⑩</p>  <p>$\varphi_A = -\varphi_B = \frac{F \cdot l^2}{16EJ}$; $y_{max} = \frac{F \cdot l^3}{48EJ}$</p>